Curriculum

August 27, 2020

Alessandro Perotti

Position:
Associate Professor Department of Mathematics University of Trento
SSD: MAT/03 – Geometria

National Scientific Qualification ('Abilitazione Scientifica Nazionale')
Full Professor ('I fascia') - ASN 2016 - Settore Conc. 01/A2 Geometria e Algebra
Associate Professor ('II fascia') - ASN 2012 - Settore Conc. 01/A2 Geometria e Algebra

Past Positions:
from 01/11/2012 to 30/9/2014 Assistant Professor Department of Mathematics at the University of Trento
from 01/11/2001 Assistant Professor Faculty of Engineering at the University of Trento
from 01/11/1998 Assistant Professor University of Milano-Bicocca
from 16/02/1989 Assistant Professor University of Milan

Education:
1/1/1985 – 1/3/1989: PhD courses ('Perfezionamento') Scuola Normale Superiore Pisa
November – December 1984: Grant INdAM a.y. 1984/85
October 1984: Laurea in matematica University of Trento

Research

My research interests are in the fields of *complex, quaternionic and Clifford analysis in one and several variables*, with applications to the *functional calculus and spectral theory* of operators on quaternionic Hilbert spaces.

In the early stages of my career I studied several classical questions of the theory of several complex variables. I studied *generalized Levi problems* on Stein manifolds and Stein spaces, questions of *extendibility of analytic objects* and *CR objects* [32], the *Cauchy-Riemann problem on piecewise smooth domains* in complex manifolds [30,31,33]. I also explored algebraic applications of integral representations. In particular, *multidimensional logarithmic residues* associated to a zero-dimensional polynomial ideal [29]. The papers [27, 28] considered again questions of *extendibility of analytic objects*. In particular, the characterization of the traces of *pluriharmonic functions* on the boundary of a bounded domain in $\mathbb{C}^n$. In [28] the *Neumann problem* for pluriharmonic functions and the *Cauchy–Riemann problem* for $(0,1)$-forms with assigned real part on the boundary were studied.

In [26] a characterization of the boundary L^2-orthogonal space to the subspace of traces of pluriharmonic functions was given by means of a tangential differential condition. In the 2-complex-dimensional case, an interesting link between these spaces and the class of *(Fueter) regular functions of a quaternionic variable* was found.

The papers [23–25] began a research program on *(Fueter) regular functions of a quaternionic variable*. The relations with the theory of holomorphic functions of two

complex variables have been analysed in more detail. The theory of quaternionic regular functions and its generalization to the theory of solutions of *Dirac operators* with values in a *Clifford algebra*, is an active research field (*Clifford analysis*), with interesting links with real and complex analysis and manifold applications.

The isomorphism of the skew field of quaternions with the space $\mathbb{C}^2$ allows to obtain new boundary *holomorphy conditions* in two complex variables. In [23,24], the regularity condition has been linked with the $\bar{\partial}$-*Neumann problem* in $\mathbb{C}^2$. Differential conditions which characterize regular homogeneous polynomials have been found and algorithms for the construction of bases of *regular spherical harmonics* have been written and implemented with software Mathematica. In [47], the relation between regular functions and the *hyperkähler structure* of $\mathbb{H}$ has been studied. Every holomorphic map w.r.t. a complex structure J_p compatible with the hyperkähler structure, induces a quaternionic regular function. It was found a criterion, based on the *energy minimizing property* of holomorphic maps, that permits to distinguish holomorphic maps in the class of regular functions.

In [20] it has been proved that the *biregular functions* (with regular inverse) are always (pseudo) biholomorphic on a dense subset of the domain w.r.t. a (not necessarily constant) almost complex structure. In the papers [22,42], the results obtained in [26–28] and new conditions on the traces L^2 of regular functions, have been applied to analyse the problem of the traces of *pluriholomorphic* functions of two complex variables. The key point here is the correspondence between the two complex components of a quaternionic regular function.

In [19,43], the techniques introduced in [22] and [42] allowed to define a *quaternionic Hilbert operator*, that generalizes the classical Hilbert transform of complex analysis. Here an essential role is played by the CR_p-structure of the boundary defined w.r.t. the complex structure J_p. A *directional Hilbert operator* H_p is defined, dependent on the direction p in S^2, linked to the Szegö projector S_p. In [21] the behaviour of regular functions w.r.t. conformal transformations of $\mathbb{R}^4$ has been studied.

In the papers [18, 48] some applications of the theory of regular functions to the *Lagrange interpolation* have been proposed. See also [11] for a different approach.

The articles [1–7,10,12,15–17,36,37,41] deal with a recent higher-dimensional function theory, the one of *slice-regular* functions. In (real) dimensions higher than two several theories introduced during the last century have met with success in matching many analytic features of the theory of holomorphic functions of one complex variable. The search for an approach better adapted to certain algebraic requirements, e.g. the inclusion of the classical theory of polynomials, led to introducing and developing, over the last decade, a new function theory. Born in the quaternionic setting, the theory has been then extended to the octonions, to the Clifford algebra setting, and more generally to real alternative *-algebras.

In [44,46], a novel approach to the *theory of slice-regular functions of several variables* has been proposed, by means of the introduction of a family of commuting complex structures on a Clifford algebra.

In [13] is given a complete characterization of slice-regularity in terms of *analyticity*, i.e. of convergent developments in series of functions. The paper [8] is devoted to the study of *singularities* of slice-regular functions in the general *-algebra setting.

In [37, 40, 45], some meeting points between the two quaternionic/Clifford function theories, the one defined by means of Fueter or Dirac differential operators and the one of slice-regular functions, have been investigated. In particular, in [37] some new

formulas relating the Cauchy-Riemann operators, the *spherical Dirac operators* and the spherical derivative of a slice function have been found. In the four-dimensional case, these results are related to *zonal harmonics* and to the *Poisson kernel* of the unit ball of $\mathbb{R}^4$.

The papers [9,14,38,39] deal with applications of the theory of slice and slice-regular functions to the *quaternionic functional calculus* and to the *spectral theory* of operators on quaternionic Hilbert spaces. These new developments have possibly applications to *Mathematical Physics (mathematical foundations of Quantum Mechanics)*. This research is related to my association to the research group TIFPA-INFN (Bell Project "Fundamental Problems in Quantum Physics").

Communications at conferences

I attended several congresses, summer schools and workshops, in Italy and abroad, giving (also invited) communications. In particular, in the last years I delivered the following talks:

- HAMS, Weimar 2020: *On some geometrical properties of slice-regular maps*
- 12th ISAAC Congress, Aveiro 2019: *An Almansi-type decomposition for slice-regular functions on Clifford algebras*.
- Joint meeting UMI-SIMAI-PTM, Wroklaw, 2018: *The analytic structure of the singular set of a slice-regular function*
- "Function Theories in Higher Dimensions", Tampere 2018: *The quaternionic Gauss-Lucas Theorem and some related results about quaternionic polynomials*
- ICCA11, Ghent 2017: *Left and right eigenvalues of quaternionic matrices are not unrelated*.
- Workshop AGACSE 2015, Barcelona: *Slice regularity and harmonicity over Clifford algebras*.
- WORKSHOP 2015 Varietà reali e complesse: geometria, topologia e analisi armonica - Pisa SNS: *Funzioni slice-regolari e armonicità*
- FIRB Meeting Firenze 2015: *Global differential equations for slice-regular functions*.
- IX ISAAC Congress, Krakow 2013: *Continuous slice functional calculus in quaternionic Hilbert spaces*.
- ICNPAA Wien 2012 - Workshop "Clifford algebras, Clifford analysis and their applications": *Slice regular functions of several Clifford variables*.
- VIII ISAAC Congress, Moskow 2011: *Slice regular functions in several variables on real alternative algebras*.
- ICCA9, Weimar 2011: *The full Dirac operator on a Clifford algebra*.

- INdAM Workshop "Different Notions of Regularity for Functions of Quaternionic Variable", Roma 2010: *Slice regularity and Fueter regularity: a 3D-meeting point for two function theories*.
- 18th International Conference on Finite or Infinite Dimensional Complex Analysis and Applications, Macau 2010: *Zero sets of polynomials and slice regular functions on Clifford algebras*.
- VII ISAAC Congress Imperial College London 2009: *A new approach to slice regularity on real algebras*.

Conference organization

- Workshop of Hypercomplex Analysis - University of Trento (Trento, September 2018).
- Member of the Scientific Committee of the Workshop INdAM "Complex function theory, its generalizations and applications" (Rome, September 12-16 2016).
- 3rd Workshop of Hypercomplex Analysis - University of Trento (Trento, December 2011).

PhD supervision

I have been the advisor of the PhD thesis of Amedeo Altavilla (2014), now a researcher (RTD/B) at University of Bari (formerly post-doc researcher at Università Politecnica della Marche and Roma Tor Vergata).

I have been member of the Examination Board for the defence of PhD-theses at the Universities of Ghent and Florence.

Research Groups and grants

I am a member of the group GNSAGA of INdAM. I am associated to TIFPA-INFN (Bell Project "Fundamental Problems in Quantum Physics") (cf. [9,14,38,39] for this research topic). I am a member of the ISAAC interest group on Clifford and Quaternionic analysis.

I took part in several co-financed projects COFIN, PRIN, FIRB of MURST/MIUR:

- PRIN 2017: *Real and Complex Manifolds: Topology, Geometry and Holomorphic Dynamics*
- PRIN 2015: *Real and Complex Manifolds: Geometry, Topology and Harmonic Analysis*
- FIRB 2012: *Geometria Differenziale e Teoria Geometrica delle Funzioni*
- PRIN 2010/11: *Varietà reali e complesse: geometria, topologia e analisi armonica*
- PRIN 2007: *Proprietà geometriche delle varietà reali e complesse*

- PRIN 2005: *Proprietà geometriche delle varietà reali e complesse*
- PRIN 2002: *Proprietà geometriche delle varietà reali e complesse*
- PRIN 2000: *Strutture speciali sulle varietà complesse. Azioni di gruppi e algebre su varietà*
- PRIN 1998: *Strutture speciali sulle varietà complesse. Azioni di gruppi e algebre su varietà*

Other activities

- I am a reviewer for MathSciNet of AMS and a referee for several international journals.
- I am a member of the Doctoral School committee of the Doctoral School in Mathematics of the University of Trento.
- I served as Vice Head of the Doctoral School for three years (2015-2018).

Publication List

Alessandro Perotti

All my recent publications are available on arXiv.

Articles in journals

[1] Alessandro Perotti. Almansi Theorem and Mean Value Formula for Quaternionic Slice-regular Functions. *Adv. Appl. Clifford Algebras*, 30, 61 (2020). DOI: http://dx.doi.org/10.1007/s00006-020-01078-4.

[2] Alessandro Perotti. Almansi-type theorems for slice-regular functions on Clifford algebras. *Complex Variables and Elliptic Equations*. Published online: 28 Apr 2020. DOI: http://dx.doi.org/10.1080/17476933.2020.1755967.

[3] Riccardo Ghiloni and Alessandro Perotti. On a class of orientation-preserving maps of $\mathbb{R}^4$. *J. of Geom. Anal.*, Published online: 24 January 2020. DOI: http://dx.doi.org/10.1007/s12220-020-00356-8.

[4] Alessandro Perotti. A four dimensional Jensen formula. *Riv. Math. Univ. Parma (N.S.)*, 11(1):139–152 (2020). http://arxiv.org/abs//1902.06485.

[5] Riccardo Ghiloni, Alessandro Perotti, and Caterina Stoppato. Division algebras of slice functions. *Proceedings of the Royal Society of Edinburgh: Section A Mathematics*, 150(4):2055–2082 (2020). Published online: 15 March 2019. DOI: http://dx.doi.org/10.1017/prm.2019.13.

[6] Riccardo Ghiloni and Alessandro Perotti. The quaternionic Gauss-Lucas theorem. *Ann. Mat. Pura Appl. (4)*, 197(6):1679–1686 (2018). DOI: http://dx.doi.org/10.1007/s10231-018-0742-z.

[7] Riccardo Ghiloni, Alessandro Perotti, and Caterina Stoppato. The algebra of slice functions. *Trans. Amer. Math. Soc.*, 369(5):4725–4762 (2017), electronically published on November 28, 2016. DOI: http://dx.doi.org/10.1090/tran/6816.

[8] Riccardo Ghiloni, Alessandro Perotti, and Caterina Stoppato. Singularities of slice regular functions over real alternative *-algebras. *Adv. Math.*, 305:1085–1130 (2017), electronically published on October 13, 2016. DOI: http://dx.doi.org/10.1016/j.aim.2016.10.009.

[9] Riccardo Ghiloni, Valter Moretti, and Alessandro Perotti. Spectral representations of normal operators via Intertwining Quaternionic Projection Valued Measures. *Rev. Math. Phys.*, 29(10):1750034, 73 (2017). DOI: http://dx.doi.org/10.1142/S0129055X17500349.

[10] Riccardo Ghiloni, Alessandro Perotti, and Vincenzo Recupero. Noncommutative Cauchy integral formula. *Complex Anal. and Oper. Theory*, 11(2):289–306 (2017). DOI: http://dx.doi.org/10.1007/s11785-016-0543-6.

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[12] Riccardo Ghiloni and Alessandro Perotti. Global differential equations for slice regular functions. *Math. Nachr.*, 287(5-6):561–573 (2014). DOI: http://dx.doi.org/10.1002/mana.201200318.

[13] Riccardo Ghiloni and Alessandro Perotti. Power and spherical series over real alternative *-algebras. *Indiana Univ. Math. J.*, 63(2):495–532 (2014). DOI: http://dx.doi.org/10.1512/iumj.2014.63.5227.

[14] Riccardo Ghiloni, Valter Moretti, and Alessandro Perotti. Continuous slice functional calculus in quaternionic Hilbert spaces. *Rev. Math. Phys.*, 25(4):1350006, 83 (2013). DOI: http://dx.doi.org/10.1142/S0129055X13500062.

[15] Riccardo Ghiloni and Alessandro Perotti. Volume Cauchy formulas for slice functions on real associative *-algebras. *Complex Var. Elliptic Equ.*, 58(12):1701–1714 (2013). DOI: http://dx.doi.org/10.1080/17476933.2012.709851.

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[21] Alessandro Perotti. Regular quaternionic functions and conformal mappings. *Cubo*, 11(1):123–143, 2009.

[22] Alessandro Perotti. Dirichlet problem for pluriholomorphic functions of two complex variables. *J. Math. Anal. Appl.*, 337(1):107–115, 2008. DOI: http://dx.doi.org/10.1016/j.jmaa.2007.03.086.

[23] Alessandro Perotti. Quaternionic regularity and the $\overline{\partial}$-Neumann problem in $\mathbb{C}^2$. *Complex Var. Elliptic Equ.*, 52(5):439–453, 2007.
DOI: http://dx.doi.org/10.1080/17476930601178392.

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DOI: http://dx.doi.org/10.1016/S1631-073X(03)00284-X.

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DOI: http://dx.doi.org/10.1080/0278107031000140916.

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DOI: http://dx.doi.org/10.5565/PUBLMAT_44200_05.

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URL: http://www.numdam.org/item?id=ASNSP_1994_4_21_4_483_0.

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Preprints

[34] Riccardo Ghiloni and Alessandro Perotti. Slice regular functions in several variables. 2020, submitted. http://arxiv.org/abs//2007.14925.

[35] Riccardo Ghiloni, Alessandro Perotti, and Caterina Stoppato. Slice regular functions and orthogonal complex structures over $\mathbb{R}^8$. 2020, submitted.

[36] Alessandro Perotti. A four dimensional Bernstein theorem. 2019. http://arxiv.org/abs//1903.03068, submitted.

Articles in books

[37] Alessandro Perotti. Slice regularity and harmonicity on Clifford algebras. In *Topics in Clifford Analysis – Special Volume in Honor of Wolfgang Sprössig*, Trends Math. Springer, Basel, 2019.
DOI: http://dx.doi.org/10.1007/978-3-030-23854-4_3.

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Conference Proceedings

[44] Riccardo Ghiloni and Alessandro Perotti. Slice regular functions of several Clifford variables. In *AIP Conf. Proc.*, volume 1493, pages 734–738. AIP, 2012. 9th International Conference on Mathematical Problems in Engineering, Aerospace and Sciences: ICNPAA 2012, Vienna, Austria, 10–14 July 2012. DOI: `https://doi.org/10.1063/1.4765569`.

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[46] R. Ghiloni and A. Perotti, Slice regularity in several variables. Progress in analysis. Proceedings of the 8th congress of the International Society for Analysis, its Applications, and Computation (ISAAC), Moscow, Russia, August 22–27, 2011. Volume 1, ISBN 978-5-209-04582-3/hbk, pages 179–186 (2012) Zbl 1298.30044.

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Trento, August 27, 2020